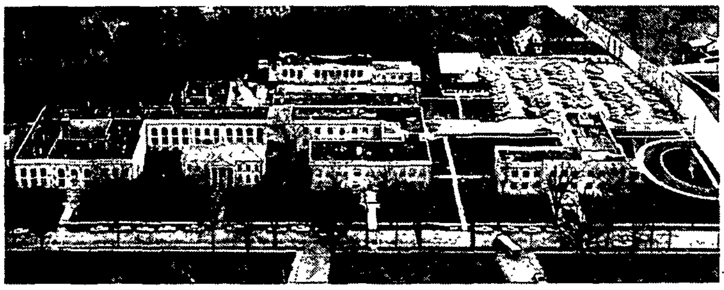

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A UNIFIED TREATMENT OF BROWNSTOCK WASHING ON ROTARY FILTERS

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A Unified Treatment of Brownstock Washing on Rotary Filters

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KEYWORDS: pulp washing, brownstock, rotary filters, equations, theory

ABSTRACT

Countercurrent washing of pulp on rotary filters is a mass transfer process which is amenable to analysis by the classical techniques of nonequilibrium staged operations. Such analysis establishes the connections among the overall-, stage-, and local-efficiencies and the underlying mass transfer rate in the wash zone. Performance and design equations result which are simple and straightforward. Associated graphical depictions of the calculation scheme serve to aid in visualizing the concepts. Currently accepted methods are seen to fit into the present analysis at different levels. The results make clear where fundamental research ought to be focused.

The purpose of this paper is to present the framework for connecting the two previously disparate methods of describing the washing of pulp. On the one hand are the fundamental treatments (1-5) which provide useful insights but are too complex for use as design or performance tools. On the other hand are practical calculation schemes (6-13) based on various concepts of perfect stages, which rely on empiricism for their utility.

The specific case considered is that of countercurrent washing on rotary filters, although the basic chemical engineering concepts utilized are applicable to other washing methods. Our aim is threefold. First, we want to relate properly defined efficiencies of washing to the underlying mass transfer rate. Second, these efficiencies are to be incorporated into the material balances to produce performance or design equations free of unnecessary algebraic complexity. Finally, these new results need to be compared with presently accepted methods.

Stagewise calculations for performance

The operation of the cascade of filters depicted in Fig. 1 can be represented in terms of the classical juxtaposition of operating and "equilibrium" lines. A material balance between the last (Mth) filter and any other (the Nth) gives the operating line:

$$\overline{X}_n = \frac{1}{N} C_{n-1} + \overline{X}_{m+1} - \frac{1}{N} C_m \tag{1}$$

Figure 1 here

The "equilibrium" line is defined as

$$\overline{X}_n = C_n \tag{2}$$

but does not represent true thermodynamic equilibrium. Henceforth, this line is called the Norden line, because any stage which obeys this relationship is "perfect" in the Norden sense. Figure 2 depicts the stepwise graphical construction to obtain the number of Norden stages necessary to produce a given wash loss.

Figure 2 here

In the classical approach to nonequilibrium staged operations, it is useful to define a stage efficiency which in this case is given by:

$$\overline{X}_n - \overline{X}_{n+1} = E_s\ (C_n - \overline{X}_{n+1}) \tag{3}$$

Figure 3 depicts the stepwise construction to obtain the required number of actual stages for a given stage efficiency.

Figure 3 here

Analytically, Eq. (3) allows the elimination of pulp concentrations from Eq. (1) and (2). The result is a second order difference equation,

$$\overline{X}_{n+1} - \left(\frac{q+1}{q}\right) \overline{X}_n + \frac{1}{q} \overline{X}_{n-1} = 0 \tag{4}$$

where

$$q = 1 + E_s\ (N-1) \tag{4a}$$

which can be solved with the following boundary conditions

$$n = m+1, \; \overline{X}_n = \overline{X}_{m+1} \tag{4b}$$

$$n = 0, \; C_n = C_o \tag{4c}$$

The solution is

$$\frac{C_m}{C_o} = L_m = 1 - \left[\frac{N\,(q^m-1)}{N\,q^m-1}\right]\left(1 - \frac{\overline{X}_{m+1}}{C_o}\right) \tag{5}$$

which is the analytic equivalent of the graphical construction in Fig. 3. This is a straightforward performance equation which gives wash loss directly in terms of wash ratio, stage efficiency and number of stages.

Minimum wash ratio

Another classical concept relates to a limiting slope of the operating line. In Fig. 2, as the wash ratio is decreased a point will be reached at which the operating line will intersect the Norden line at C_o. The slope of this line corresponds to the minimum wash ratio, N_m, which is defined as the value of the wash ratio at which an infinite number of Norden stages would be required to attain the specified wash loss.

From Fig. 4:

$$N_m = \frac{1 - L_m}{1 - \dfrac{\overline{X}_{m+1}}{C_o}} \tag{6}$$

Figure 4 here

As we will see, the ratio of Norden stages to actual stages is a well defined overall efficiency which remains finite. If the number of required Norden stages becomes infinite so would the required actual stages. N_m can thus be interpreted as the wash ratio at which an infinite number of actual stages would be required.

Stagewise calculations for design

In practice, a wash ratio can be specified as some multiple of N_m, say

$$N = \alpha N_m \tag{7}$$

Using Eq. (6) and (7), Eq. (5) can be inverted to obtain

$$m = \frac{\ln\left[\dfrac{\alpha - 1}{\alpha(1 - N_m)}\right]}{\ln q} \tag{8}$$

which is a convenient explicit relationship for the actual required number of filters.

Mass transfer in the wash zone

In the wash zone, mass transfer occurs under the influence of spatially varying concentration gradients in the presence of possible fluid dispersion within the porous fibrous mat. The pulp mat moves in a transverse direction with respect to the flow of wash liquor (Fig. 5). This mode of contact is similar to the flow of the two phases involved on a tray of a distillation column. Although in washing, the phases and the physics of contact are different, the analogy is useful, and much of the analysis of mass transfer on a distillation tray can be adapted to the wash zone (14).

Figure 5 here

In fact, if a local efficiency is defined by

$$X_n - \overline{X}_{n+1} = E\ (C - \overline{X}_{n+1}) \tag{9}$$

then, neglecting dispersion and assuming transverse plug flow of pulp, it can be shown (14) that

$$E = 1 - e^{-\eta} \tag{10}$$

where

$$\eta = \frac{kaH}{\hat{W}} \tag{10a}$$

which is the number of transfer units and can be interpreted as a dimensionless mass transfer coefficient. Unlike other efficiencies, the local efficiency, E, must be between zero and one. E is a true measure of the mass transfer efficiency of the process, and, through Eq. (10) [or more complicated expressions if dispersion is important (14)], it is directly related to basic mass transfer parameters.

Equation (9) can be incorporated into a differential material balance which can be integrated across the wash zone to give (14):

$$C_n - \overline{X}_{n+1} = e^{-EN}\ (b_n - \overline{X}_{n+1}) \tag{11}$$

Single stage material balances

In the continuous rotary filter, depicted in Fig. 6, the feed pulp is reslurried with a portion of the exiting wash liquor. Cake is continuously formed on the rotating drum and is dewatered and washed before removal.

Figure 6 here

A material balance over the entire stage gives

$$C_{n-1} + N\overline{X}_{n+1} = C_n + N\overline{X}_n \tag{12}$$

A balance over the reslurrying tank gives

$$C_{n-1} + R\overline{X}_n = (1 + R)\ b_n \tag{13}$$

These last two equations can be combined with Eq. (3) and (11) to yield a useful relationship for the stage efficiency:

$$E_s = \frac{(1 + R)e^{EN} - 1}{R + N} \tag{14}$$

Figure 7 is a plot of Eq. (14) for typical recycle and wash ratios. Note that, even when E = 0, E_s has a sizeable value. This simply means that, even if the rate of mass transfer were zero, the stage would provide separation by pure dilution. Also when $E_s = 1$, the stage is by definition a Norden stage; and, in this case, the actual local efficiency is far less than one. In fact, over a wide range of local efficiencies, the stage efficiency is greater than one. Stages in this range actually perform better than the so-called "perfect" Norden stage.

Figure 7 here

Comparison with Norden method

In the present context, a Norden stage corresponds to $E_s = 1$. In this case, according to Eq. (4a), q = N, and, from Eq. (8) we get

$$m_N = \left[\frac{\ln\left(\dfrac{\alpha - 1}{\alpha\ (1 - N_m)}\right)}{\ln\ N}\right] \tag{15}$$

which can be interpreted as the "minimum" number of required stages (in the Norden sense).

The connection of the present analysis with the Norden method is made explicit by defining an overall efficiency as the ratio of the required number of Norden ("perfect") stages to the required number of actual stages:

$$E_o = \frac{m_N}{m} \tag{16}$$

Then, from Eq. (8) and (15):

$$E_o = \frac{\ln q}{\ln N} \tag{17}$$

which, through Eq. (4a), relates the overall (Norden) efficiency to the stage efficiency. In turn the stage efficiency is related to the local efficiency through Eq. (14).

Figure 8 is a plot of Eq. (17) for typical values of the wash ratio. If the wash ratio is exactly one, $E_o = E_s$. Otherwise these two efficiencies differ in value. Note that, since according to Fig. 7, E_s can be greater than one, so can E_o exceed one. In these cases fewer than the calculated numbers of Norden stages are required.

Figure 8 here

Comparison with displacement ratio method

The displacement ratio of Perkins <u>et al</u>. (<u>13</u>) is defined as

$$\text{D.R.} = \frac{b_n - C_n}{b_n - \bar{X}_{n+1}} \tag{18}$$

so that comparison with Eq. (11) gives the relationship of D.R. to the local efficiency:

$$\text{D.R.} = 1 - e^{-EN} \tag{19}$$

This relation can be compared to the theoretical expression of Perkins *et al.* (13):

$$\text{D.R.} = 1 - \left(\frac{Z}{Z + N}\right)^{Z} \tag{20}$$

which is obtained by assuming that the showers result in a series of perfect dilutions and thickenings.

Equations (19) and (20) are comparable if Z is considered as a mixing parameter (rather than the actual number of showers). Then

$$E = \frac{Z}{N} \ln \left(1 + \frac{N}{Z}\right) \tag{21}$$

and from this it follows directly that

$$\lim_{Z \to \infty} E = 1 \tag{21a}$$

and

$$\lim_{Z \to 0} E = 0 \tag{21b}$$

So Eq. (19) can be considered the distributed parameter representation and Eq. (20) the lumped parameter representation of the same phenomenon.

Washing efficiency

A variety of washing efficiencies abound in the literature. Here three efficiencies are singled out, each rationally defined: the local, stage and overall efficiencies. These form a hierarchy beginning with the most fundamental, the local efficiency which is directly related to the rate of the underlying separation process. Next is the stage efficiency which is most useful in characterizing the performance of a given filter stage. Finally, there is the overall efficiency which contains no detail about the process, but is simply a ratio of numbers of hypothetical to actual stages. The three efficiencies are related one to another without ambiguity. The currently accepted practical calculational schemes are seen to fit into this hierarchical structure at different levels. On the one hand, the Norden method is essentially an overall efficiency method. On the other, the displacement ratio method is really at the level of the fundamental local efficiency.

For either design or performance calculations, the stage efficiency is the most useful concept. The relationships developed here between the stage efficiency and the local efficiency (and hence the dimensionless mass transfer coefficient) provide the framework for connecting the fundamental and practical treatments of pulp washing.

Conclusion

The observation made here that the local washing efficiency (whose value for given wash and recycle ratios dictates the stage and overall efficiencies) is governed by a dimensionless mass transfer coefficient suggests where fundamental investigations should be targeted. The mass transfer coefficient must depend on pulp mat structure, the Reynolds number in the wash zone and the Schmidt number of the solute in question.

This treatment of washing on rotary filters also relieves some of the apparent algebraic complexity of previous approaches to performance calculations. In particular, Fig. 3 provides the engineer with an easily understandable picture of the essence of countercurrent filter calculations. Furthermore, although Eq. (5) requires a single stage efficiency applicable to all stages in a cascade, the construction of Fig. 3 is easily modified if an individual value of E_s is available for each stage.

Finally, these results apply only to the washable portions of each solute. A correction for sorption effects needs to be applied if specific information suggests that the level of bound solute is significant.

NOMENCLATURE

a = effective mass transfer area per unit volume

b_n = solute concentration in the reslurried feed to the Nth filter

C = solute concentration in the cake within the wash zone

C_n = concentration of solute in the cake exiting from the Nth filter

C_o = solute concentration in the pulp entering the first stage of the cascade

D.R. = displacement ratio

E = local efficiency

E_s = stage efficiency

E_o = overall efficiency

H = thickness of filter cake

k = mass transfer coefficient

L_m = overall system loss ratio

m = total stages in the cascade

N = the wash ratio = W/Q

N_m = minimum wash ratio

n = general stage index

q = defined by Eq. (4a)

Q = volumetric rate of liquor held in filter cake

R = the recycle ratio = S/Q

S = volumetric rate of wash liquor recycle

$\hat{W}$ = effective velocity of wash liquor in the wash zone

W = volumetric wash flow rate

X = solute concentration in the wash liquor within the wash zone

$\overline{X}_{n+1}$ = solute concentration in the wash liquor feed to the Nth stage

X_n = solute concentration in the wash liquor leaving any point in the wash zone of the Nth stage

Z = number of showers (or mixing parameter)

α = N/N_{min}

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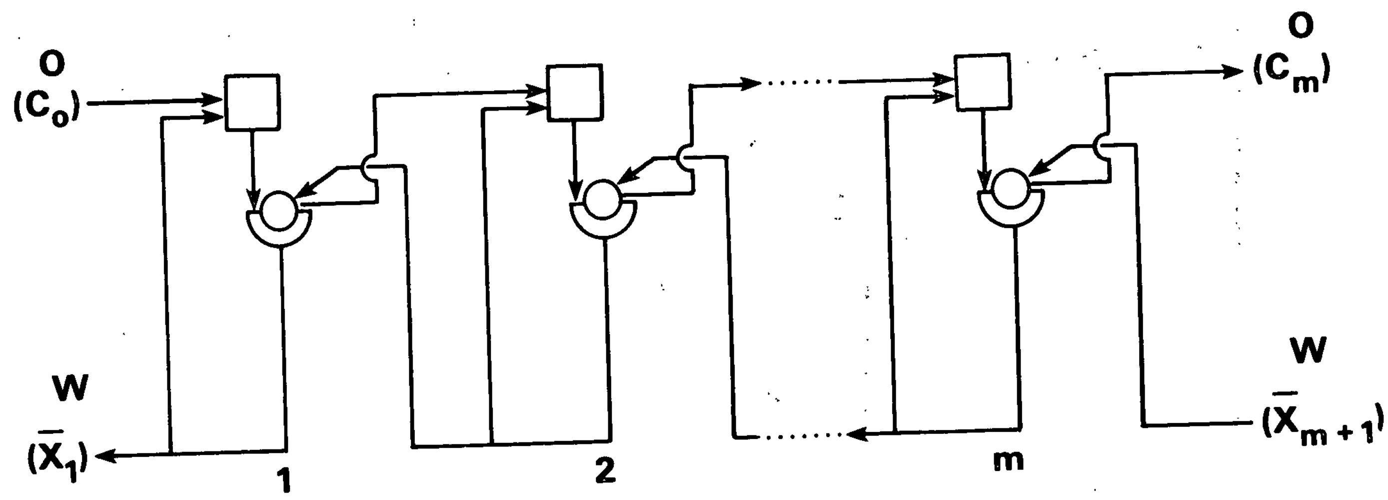

Figure 1. Countercurrent cascade of filters.

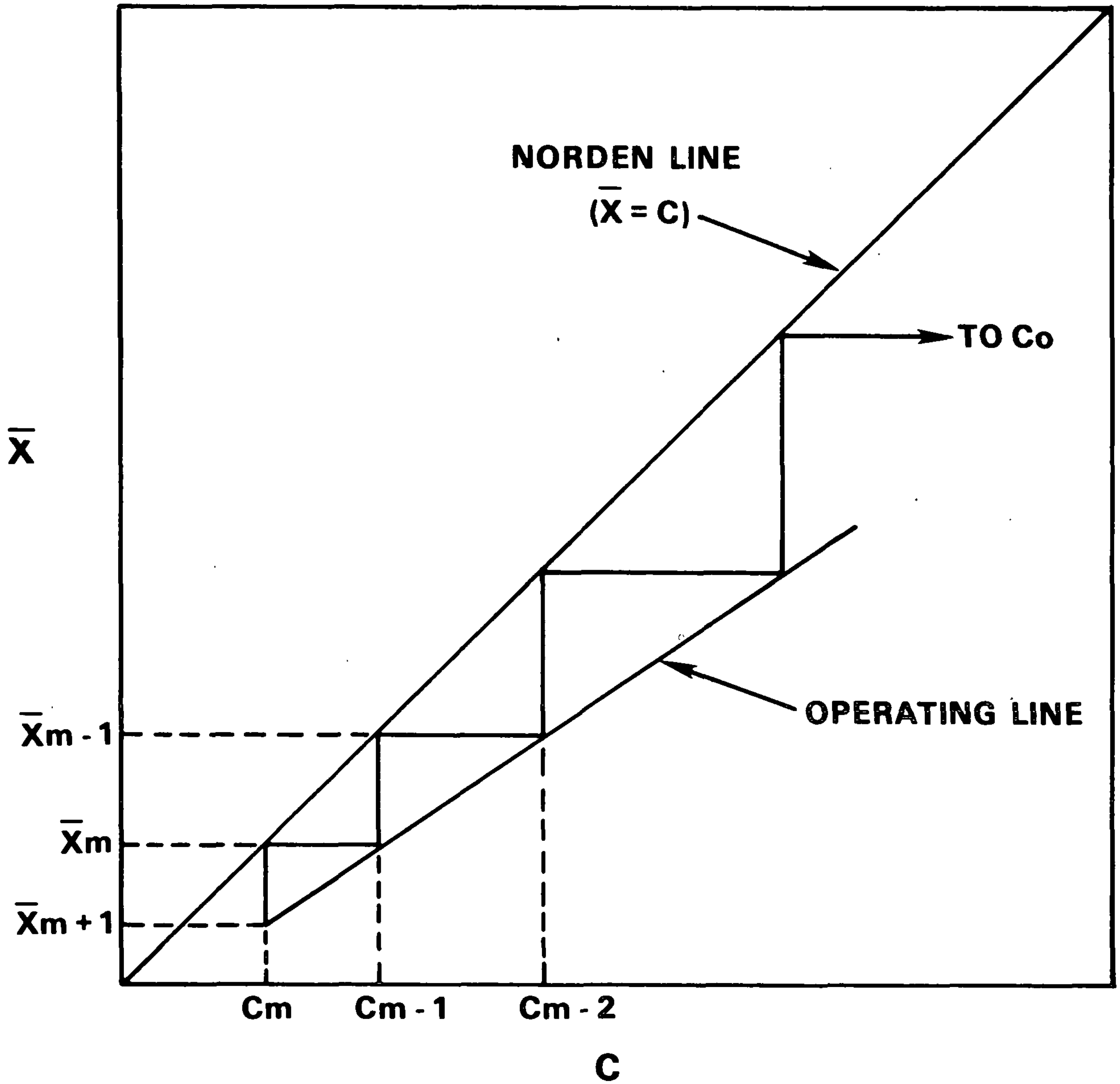

Figure 2. Calculation for Norden stages.

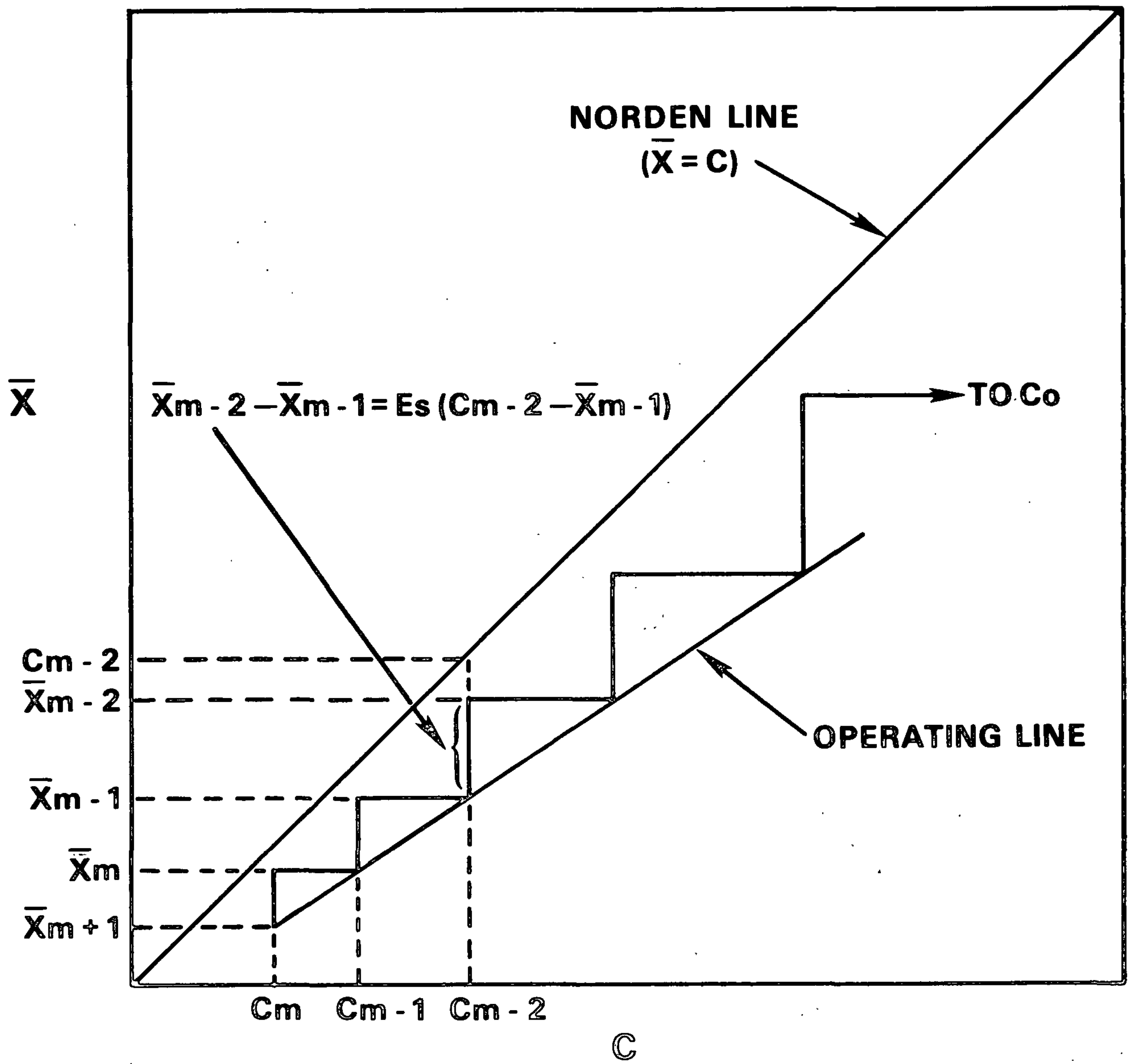

Figure 3. Calculation for actual stages.

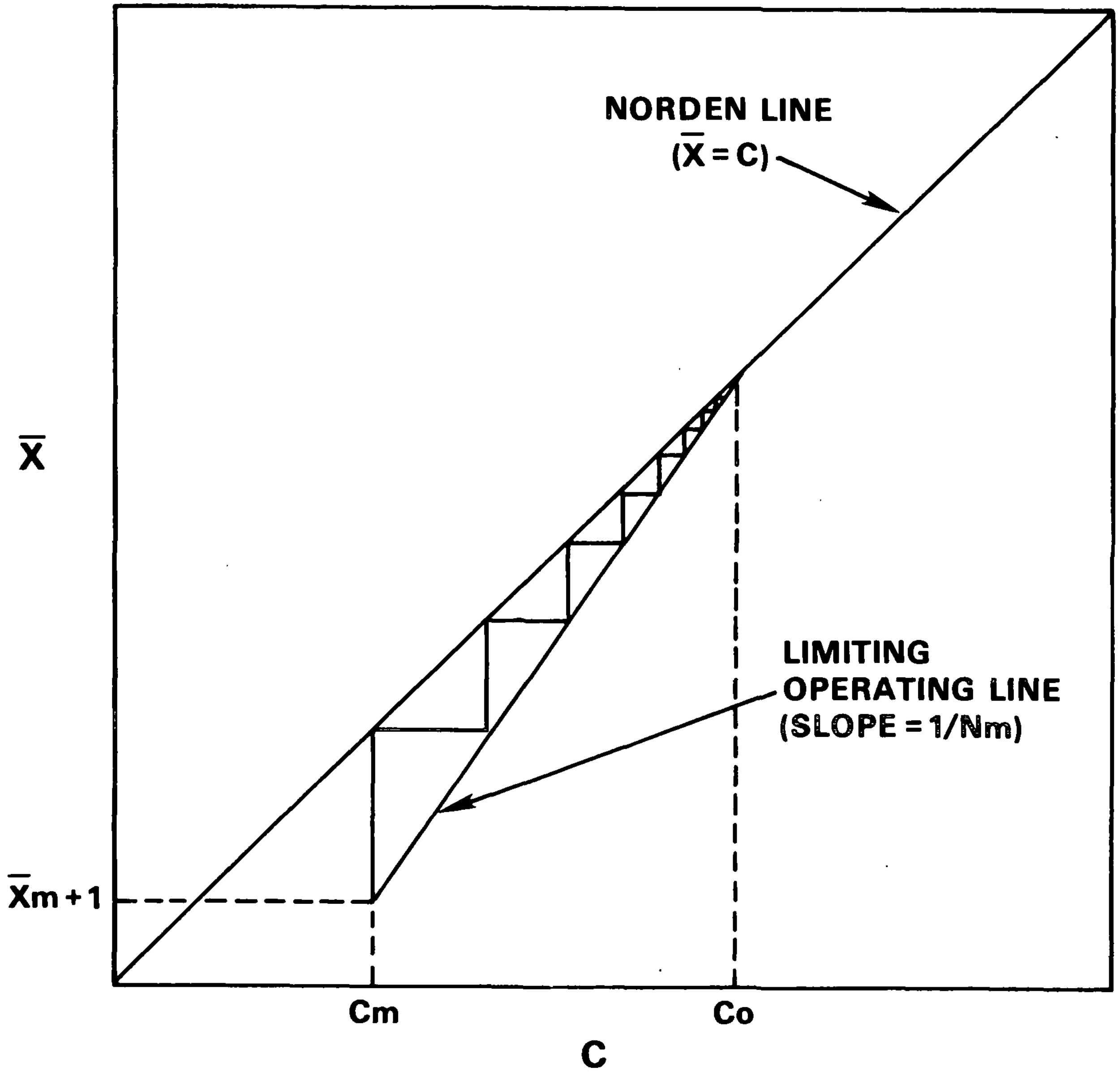

Figure 4. The minimum wash ratio.

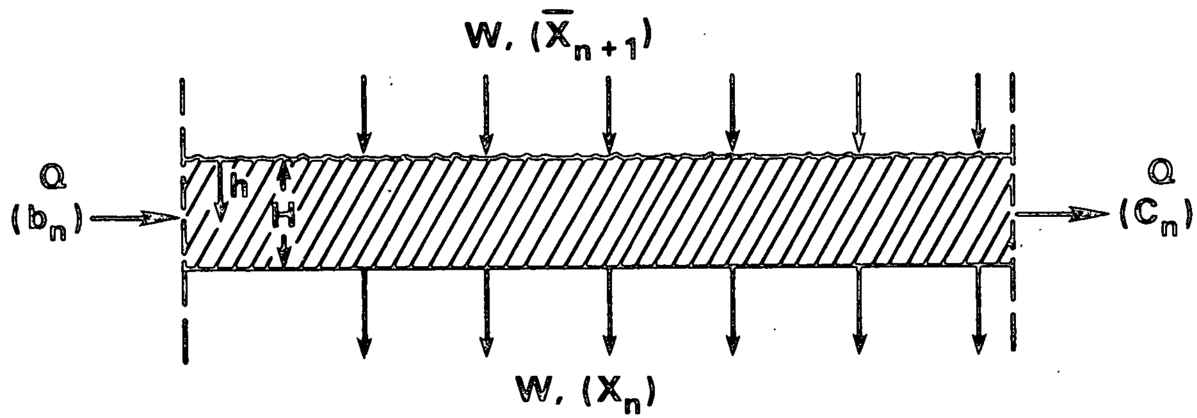

Figure 5. Schematic of the wash zone.

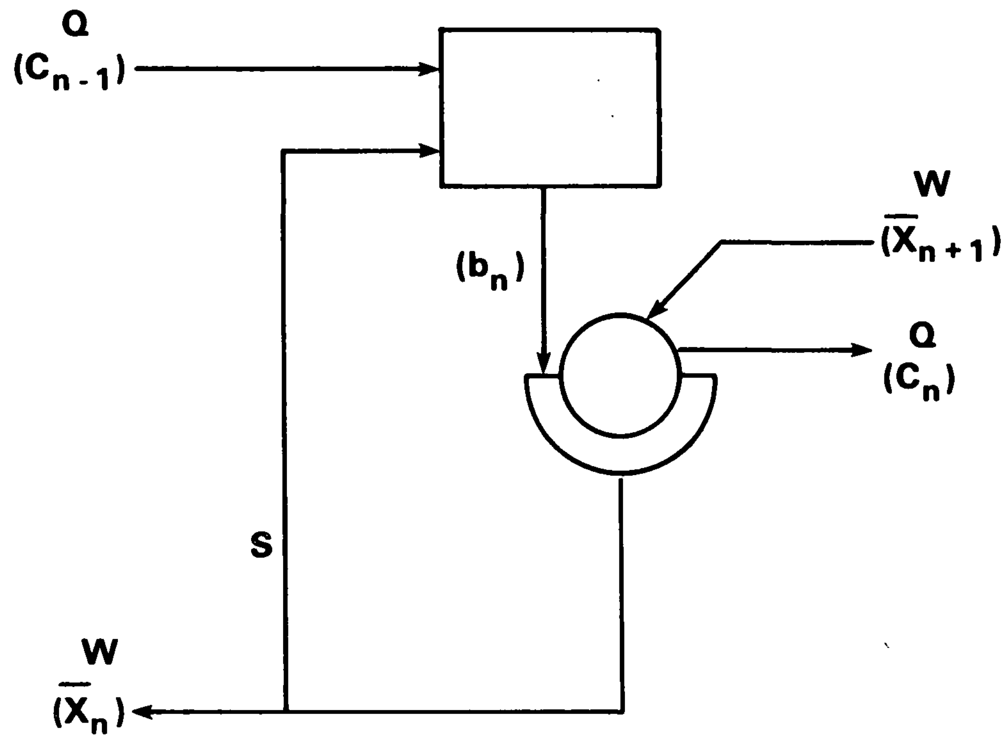

Figure 6. The continuous rotary filter.

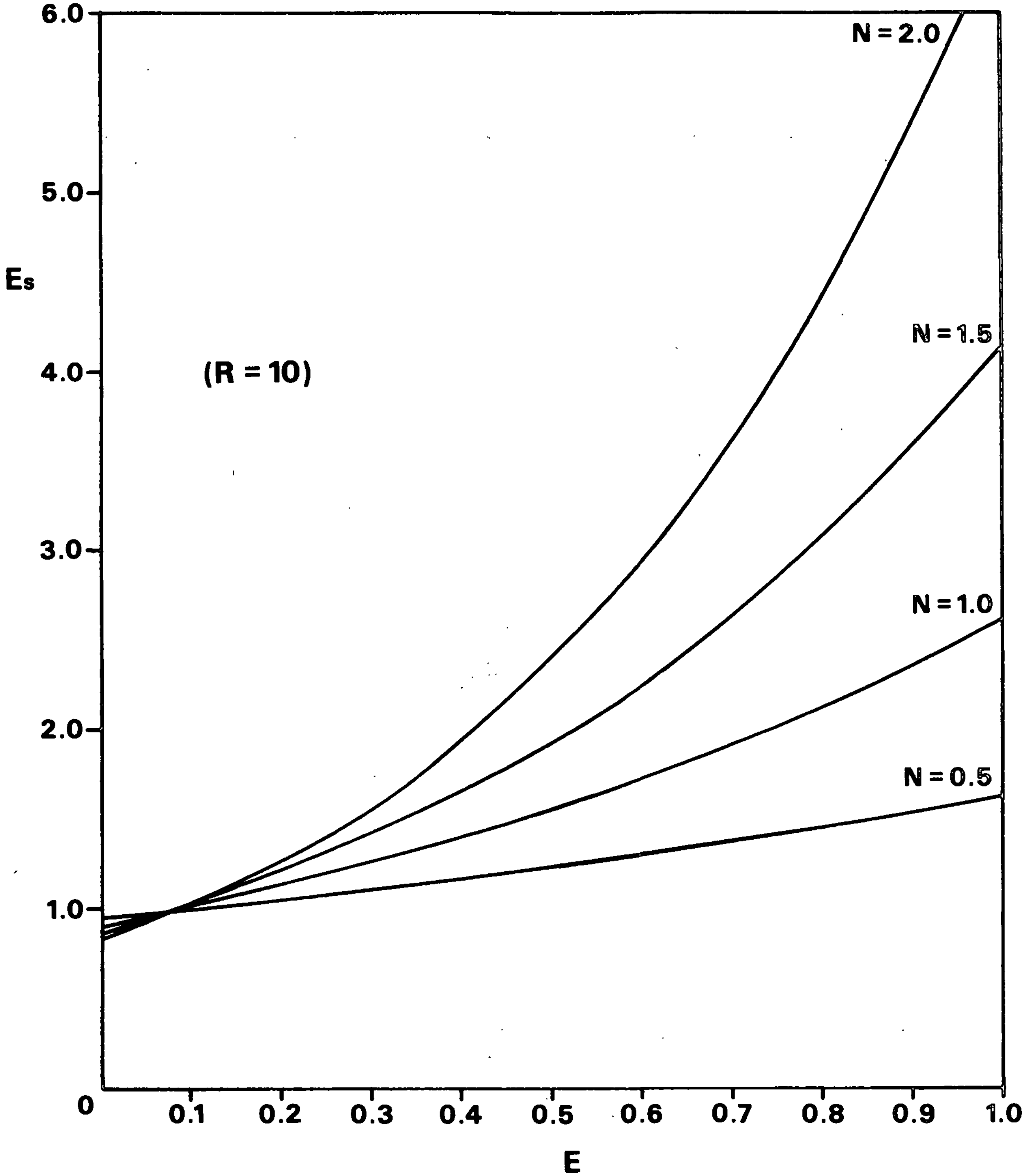

Figure 7. Stage efficiency vs. local efficiency.

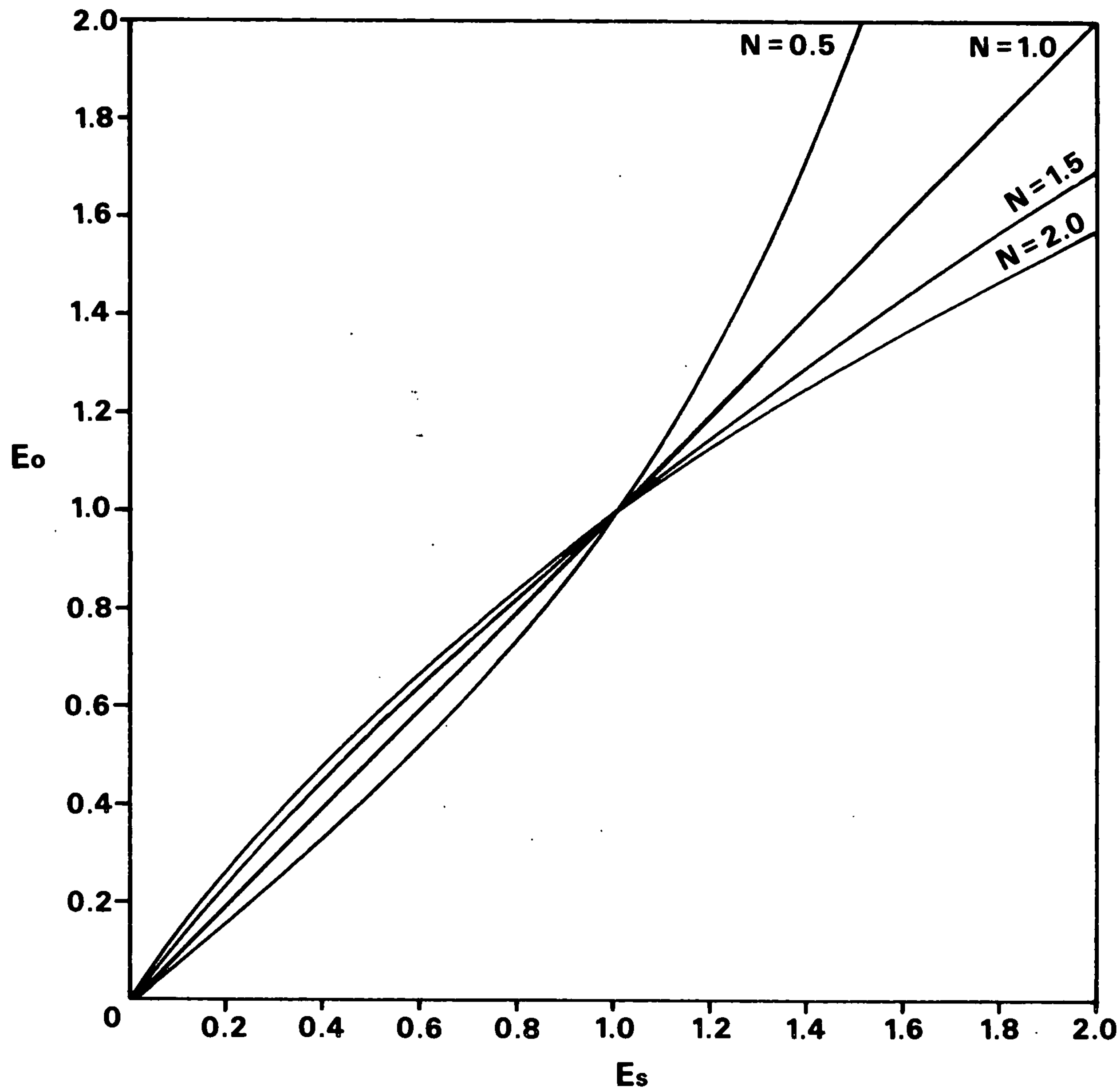

Figure 8. Overall efficiency vs. stage efficiency.

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